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Opposing mixed convection along vertical isothermal moving bodies

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Abstract

The steady laminar boundary layer flow along vertical isothermal plates and cylinders is studied in this paper. The bodies are moving upwards while their temperature is lower than the ambient environment. This is an opposing mixed convection problem. The results are obtained with the direct numerical solution of the boundary layer equations. The velocity profiles take some interesting shapes that have not been observed until now.

Keywords: Opposing mixed convection; Moving plate; Moving cylinder

1. Introduction

The convective heat transfer which is neither dominated by pure forced nor pure free convection, but is rather a combination of the two, is referred to as combined or mixed convection. In the case where the fluid is externally forced to flow in the same direction as the buoyancy force, the phenomenon is termed assisting mixed convection. In the case where the fluid is externally forced to flow in the opposite direction to the buoyancy force, the phenomenon is termed opposing mixed convection. The external force may be produced either by the motion of the body or by an external free stream.

The most representative studies for mixed convection along a moving plate are those of Moutsoglou and Chen (1980), Ramachandran et al. (1987) and Lin and Hoh (1997). However, velocity and temperature profiles for the opposing mixed convection are rare in the literature. In the work of Moutsoglou and Chen (1980) only one velocity profile is given for $\Omega = -1$ and Prandtl numbers 0.7 and 7.0. For the cylinder case it appears that the only work concerning mixed convection over a vertical isothermal moving cylinder is that by Takhar et al. (2000). The objective of the present note is to present velocity and temperature profiles for the opposing mixed convection because, as will be shown later, the velocity profiles take some interesting shapes that have not been observed until now.

2. The mathematical model

Consider the laminar flow along a vertical isothermal cylinder moving upwards with u and v denoting respectively the velocity components in the x and r direction, where x is vertically upwards and r is the coordinate perpendicular to x. The flow is assumed to be steady, of the boundary-layer type. The governing equations of this flow with Boussinesq approximations are

continuity equation:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{1}{\rho r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right) - \frac{\rho - \rho_a}{\rho}g \tag{2}$$

energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{1}{\rho c_p r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) \tag{3}$$

In the above equations T is the fluid temperature, ρ and ρ_a are the local and ambient fluid density, μ is the dynamic viscosity, k is the thermal conductivity and c_p is

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the specific heat. In the present paper the working fluid is water. However, the results which will be shown later, are valid for any fluid. The density of water is a function of temperature, salinity and pressure. In this paper the international equation of state for seawater (Fofonoff, 1985) known from oceanography is used for the calculation of density. The dynamic viscosity, thermal conductivity and specific heat have been calculated using data given by Kukulka et al. (1987). The following boundary conditions were applied:

at $r = r_0$: $u = u_0, \quad v = 0, \quad T = T_0$

as $r \to \infty$: $u = 0, \quad T = T_a$

where r_0 is the cylinder radius, u_0 is the cylinder velocity, T_0 is the cylinder temperature and T_a is the ambient temperature.

Eqs. (1)–(3) form a parabolic system and are solved directly, without any transformation, using the method described by Patankar (1980). The finite difference method is used with primitive coordinates x, r and a space marching procedure is used in the x direction with an expanding grid. A detailed description of the solution procedure may be found in Pantokratoras (2002).

As was mentioned before the only work concerning mixed convection over a vertical isothermal moving cylinder is that by Takhar et al. (2000). In this work the results are presented in figures and no numerical values are given. For that reason the accuracy of the present method was tested comparing the results with those of the pure forced convection and pure free convection. The accuracy of the present method for pure free convection has been tested in another paper (Pantokratoras, 2000). For pure forced convection the present method has been tested with the results of Karnis and Pechoc (1978). This comparison was satisfactory.

3. Results and discussion

It has been generally recognized that the governing parameters for this problem is the pseudo-similarity variable η, the curvature parameter ξ and the buoyancy parameter Ω defined as (Takhar et al., 2000)

$$\eta = \frac{r^2 - r_0^2}{4r_0}\left[\frac{u_0}{\nu_f x}\right]^{1/2} \tag{4}$$

$$\xi = \frac{4}{r_0}\left[\frac{\nu_f x}{u_0}\right]^{1/2} \tag{5}$$

$$\Omega = Gr_x/Re_x^2 \tag{6}$$

where Re_x is the local Reynolds number and Gr_x is the local Grashof number defined as

$$Re_x = \frac{u_0 x}{\nu_f} \tag{7}$$

$$Gr_x = \frac{g x^3}{\nu_f^2}\frac{\rho_a - \rho_0}{\rho_a} \tag{8}$$

where ρ_0 is the water density at the cylinder surface. The kinematic viscosity is calculated at film temperature $(T_0 + T_a)/2$. Pure forced convection exists as a limit when Ω goes to zero and pure free convection is reached when Ω becomes infinite. The buoyancy parameter Ω represents the ratio of buoyancy forces to inertial forces and when $\Omega = 1$ the two forces are of comparable order. Values of Ω for opposing mixed convection are negative. When the cylinder radius becomes infinite the curvature parameter becomes zero ($\xi = 0$) and the cylinder transforms to a flat plate. In the present paper the flat plate is treated as a special case of the more general cylinder problem. For the flat plate case the buoyancy parameter Ω remains the same while the pseudo-similarity variable η takes the form

$$\eta = y\left[\frac{u_0}{\nu_f x}\right]^{1/2} = \frac{y}{x} Re_x^2 \tag{9}$$

The most important quantities in mixed convection problem are the wall heat transfer and the wall shear stress which are directly related to temperature and velocity profiles.

In Fig. 1 some representative velocity and temperature profiles are shown for an upward moving cylinder in water with surface temperature 37 °C and ambient water temperature 40 °C while in Fig. 2 velocity and temperature profiles are shown for an upward moving plate in water with the same temperatures. In Fig. 3 some representative velocity and temperature profiles are shown for an upward moving plate in water with plate temperature 17 °C and ambient water temperature 20 °C. Figures for other temperature ranges have been produced for both cylinder and plate but are not presented because the qualitative characteristics are the same. The solid lines correspond to velocity and dashed lines to temperature. The case with $\Omega = 0$ represents a pure forced convection situation which can be reached either when the Grashof number is zero or when the Reynolds number is infinite. In the present work this situation has been reached by setting the gravity acceleration equal to zero (zero Grashof number). In that case the last term in the momentum equation is zero and, although there is a temperature difference between the plate and the ambient fluid, the buoyancy force is zero. Taking into account the above figures the following conclusions can be drawn for both bodies:

1. The velocity profiles do not follow the usual monotonic variation across the boundary layer (continuous decrease of velocity with increasing η). We see that as the buoyancy parameter decreases the velocity profiles show first a local minimum and then a local maximum.

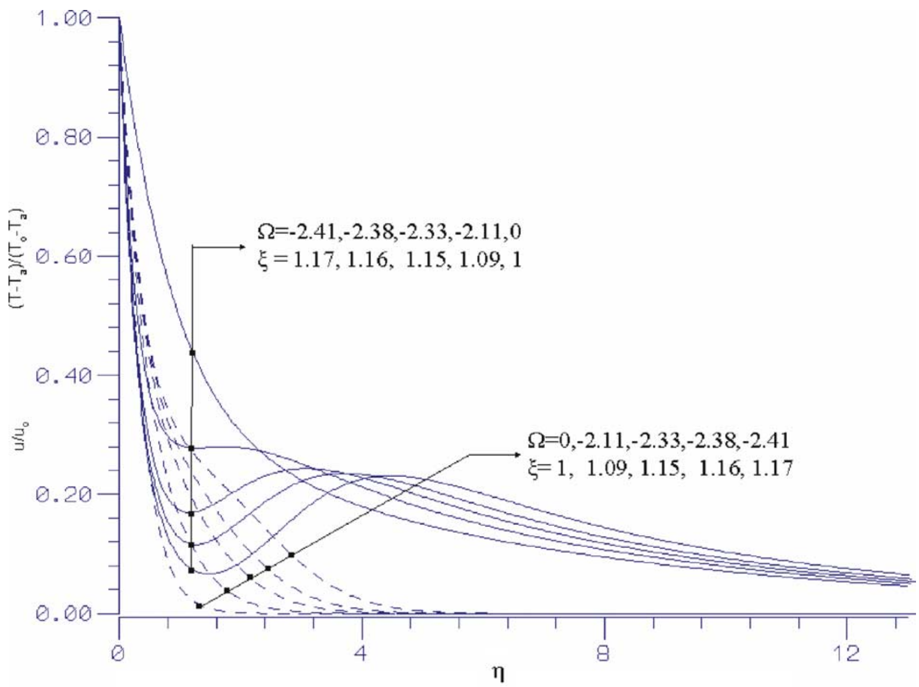

Fig. 1. Representative velocity and temperature profiles for an upward moving cylinder in water with $T_0 = 37$ °C, $T_a = 40$ °C and different values of parameters Ω and ξ (film Prandtl number 4.09). Solid lines correspond to velocity and dashed lines to temperature.

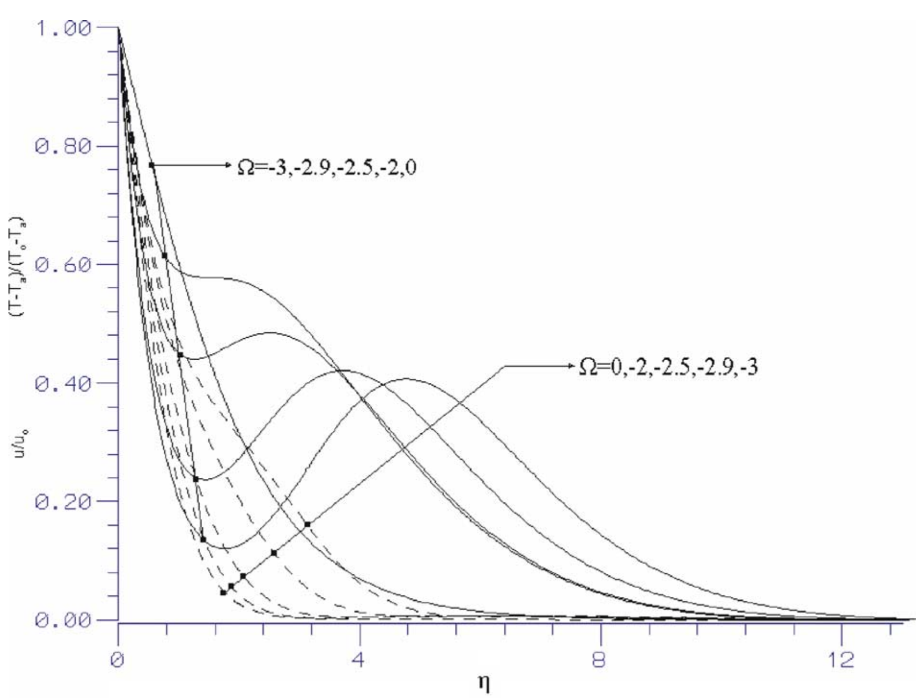

Fig. 2. Representative velocity and temperature profiles for an upward moving plate in water with $T_0 = 37$ °C, $T_a = 40$ °C and different values of parameter Ω (film Prandtl number 4.09). Solid lines correspond to velocity and dashed lines to temperature.

2. The location of the local minimum and the local maximum shifts to higher η as Ω decreases.
3. The velocity boundary layer thickness increases as Ω decreases.

It is the first time in the literature that these velocity profiles are being observed. The explanation can be drawn from Fig. 4. In this figure the vertical velocity profile for $\Omega = -3$ (taken from Fig. 2) is shown together

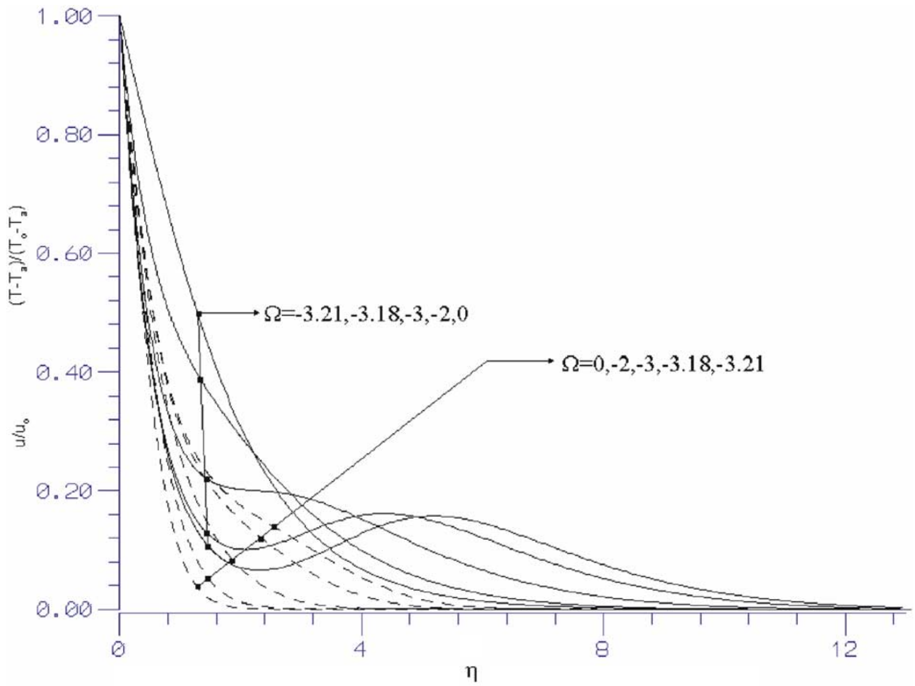

Fig. 3. Representative velocity and temperature profiles for an upward moving plate in water with $T_0 = 17$ °C, $T_a = 20$ °C and different values of parameter Ω (film Prandtl number 7.27). Solid lines correspond to velocity and dashed lines to temperature.

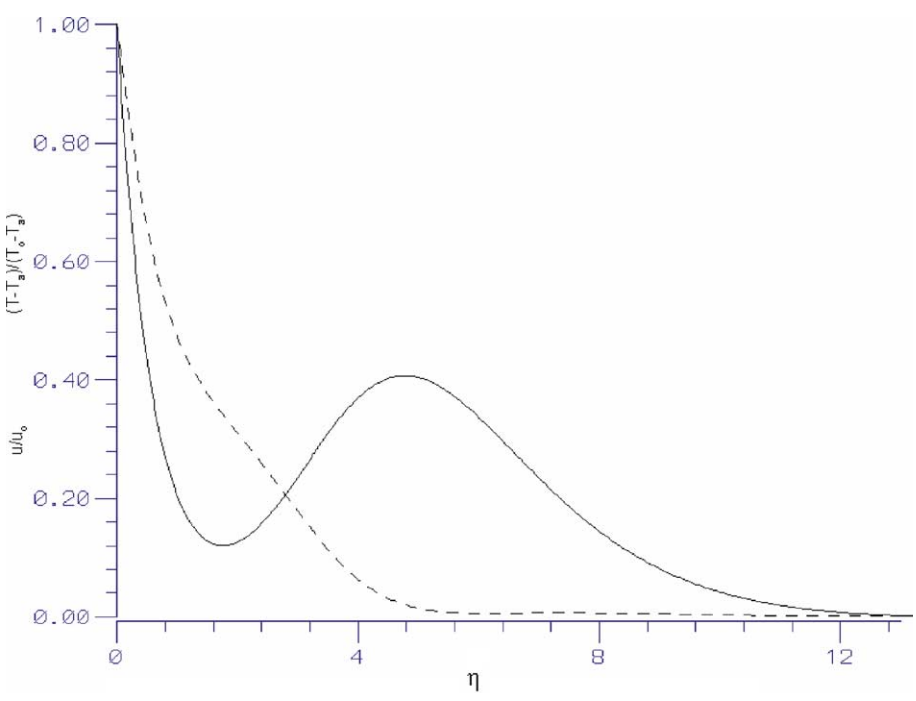

Fig. 4. Velocity and temperature profiles for an upward moving plate in water with $T_0 = 37$ °C, $T_a = 40$ °C and $\Omega = -3$. Solid line corresponds to velocity and dashed line to temperature.

with the corresponding temperature profile (dashed line). It is seen that the thermal boundary layer extends up to $\eta = 5$ whereas the velocity boundary layer extends up to $\eta = 13$. The temperature varies from 37 to 40 °C up to $\eta = 5$ and remains constant (40 °C) from $\eta = 5$ to 13. It is obvious that from $\eta = 5$ to 13 the opposing buoyancy force is zero. It is remarkable that the local velocity maximum occurs at the point where the boundary layer temperature becomes equal to ambient temperature and the opposing buoyancy force becomes zero. In the region from $\eta = 0$ to 2 the velocity decreases continuously because the opposing buoyancy force is still large as can be seen from the temperature profile. In the region from $\eta = 2$ to 5 the opposing buoyancy force decreases significantly and finally becomes zero at $\eta = 5$. It is reasonable that in this region the velocity starts to increase and becomes maximum at the point where the opposing buoyancy force becomes zero. The velocity from $\eta = 5$ to 13 is produced only by viscous forces. The fluid motion in this region is similar to the classical fluid mechanics problem of a vertical plate (in our case at $\eta = 5$) moving upwards with constant velocity in an infinite isothermal viscous fluid. It should be also noted that the temperature profile consists of a straight line in the region from $\eta = 1.5$ to 3.5. This linear part of the temperature profile appears also in the other figures.

Another question that arises from the above figures is why the velocity takes values that are higher than those of the velocity of pure plate movement ($\Omega = 0$). The explanation is the following: In forced convection along a plate advection and diffusion contradict each other. When the external velocity (free stream or plate velocity) along the plate increases the diffusion decreases and the momentum boundary layer decreases (the velocity profile becomes narrower). In contrary, a lower momentum flux by advection will allow the diffusion effects to further penetrate in the y-direction, thickening the momentum boundary layer. In our case the opposing buoyancy decreases the advection near the plate (it is clear from the figures), the diffusion increases and the momentum boundary layer thickness increases, too. Apparently, the momentum boundary layer thickness, which corresponds to the opposing buoyancy case, will be larger than that of zero buoyancy ($\Omega = 0$). This means that, away from the plate, the velocity profiles of the opposing buoyancy cases should lie outside the profile of zero buoyancy ($\Omega = 0$) as it is shown in Figs. 1–3.

The above behavior of temperature and velocity profiles have been also observed in air. It is advocated here that the above characteristics of the temperature and velocity profiles will appear in any fluid and in any kind of vertically moving body because these phenomena are caused by the interaction between the opposing buoyancy and inertial forces. Consequently a complete study of opposing mixed convection in moving bodies should include velocity and temperature profiles, results for the velocity minima and maxima and the velocity and temperature boundary layer thickness. It should also be fruitful to investigate the possible influence of these maxima and minima on the wall heat transfer and wall shear stress reported in the previous studies.

4. Conclusions

A numerical solution procedure has been employed to study the laminar opposing mixed convection along vertical isothermal moving bodies. The major findings from the present study can be summarized as follows:

1. The velocity profiles do not follow the usual monotonic variation across the boundary layer. As the buoyancy parameter decreases the velocity profiles show first a local minimum and then a local maximum.
2. The location of the local minimum and the local maximum shifts to higher distances from the plate as the buoyancy parameter decreases.
3. The velocity boundary layer thickness increases as the buoyancy parameter decreases.
4. The velocity may reach values that are higher than those which correspond to plate movement without buoyancy. This is due to opposing action of advection and diffusion along the plate.

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